J. Fluid Mech. (1992), *vol.* 241, *pp.* 503–523
Printed in Great Britain

Stabilization and destabilization of turbulent shear flow in a rotating fluid

By D. J. TRITTON†
Department of Physics, University of Newcastle upon Tyne, NE1 7RU, UK

(Received 3 January 1990 and in revised form 22 January 1992)

We consider turbulent shear flows in a rotating fluid, with the rotation axis parallel or antiparallel to the mean flow vorticity. It is already known that rotation such that the shear becomes cyclonic is stabilizing (with reference to the non-rotating case), whereas the opposite rotation is destabilizing for low rotation rates and restabilizing for higher. The arguments leading to and quantifying these statement are heuristic. Their status and limitations require clarification. Also, it is useful to formulate them in ways that permit direct comparison of the underlying concepts with experimental data.

An extension of a displaced particle analysis, given by Tritton & Davies (1981) indicates changes with the rotation rate of the orientation of the motion directly generated by the shear/Coriolis instability occurring in the destabilized range.

The 'simplified Reynolds stress equations scheme', proposed by Johnston, Halleen & Lezius (1972), has been reformulated in terms of two angles, representing the orientation of the principal axes of the Reynolds stress tensor (α_a) and the orientation of the Reynolds stress generating processes (α_b), that are approximately equal according to the scheme. The scheme necessarily fails at large rotation rates because of internal inconsistency, additional to the fact that it is inapplicable to two-dimensional turbulence. However, it has a wide range of potential applicability, which may be tested with experimental data. α_a and α_b have been evaluated from numerical data for homogeneous shear flow (Bertoglio 1982) and laboratory data for a wake (Witt & Joubert 1985) and a free shear layer (Bidokhti & Tritton 1992). The trends with varying rotation rate are notably similar for the three cases. There is a significant range of near equality of α_a and α_b. An extension of the scheme, allowing for evolution of the flow, relates to the observation of energy transfer from the turbulence to the mean flow.

1. Introduction

This paper aims to clarify the processes involved in a turbulent shear flow in a rotating fluid. The main part is a reformulation of ideas originally introduced by Johnston, Halleen & Lezius (1972) to explain the stabilizing or destabilizing influence of rotation. This reformulation provides some additional understanding of their scheme and, more particularly, enables a more direct comparison of experimental data (laboratory or numerical experiments) with its predictions. It was motivated by the interpretation of data in an accompanying paper (Bidokhti & Tritton 1992) on a free shear layer in a rotating fluid. However, the ideas are of sufficient generality that it is useful to separate them from that particular context, and apply them also

† Present address: Institut de Mécanique de Grenoble, Domaine Universitaire, B. P. 53X, 38041 Grenoble, France.

to other flows. Additionally, it seems worthwhile expanding the discussion a little to relate the ideas to other discussions of this topic, as explained at the end of this section.

The specific configuration under consideration is an approximately unidirectional shear flow in a rotating reference frame, with the vorticity associated with the shear parallel or antiparallel to that of the system rotation. We choose Cartesian coordinates, so that the mean flow is in the x-direction with speed varying in the y-direction, $\boldsymbol{U} = (U(y), 0, 0)$. The system rotates with angular velocity Ω about the z-axis. The shear vorticity is

$$\zeta = -\mathrm{d}U/\mathrm{d}y \tag{1}$$

and the effect of rotation is indicated by the ratio of the background vorticity to this:

$$S = 2\Omega/\zeta \tag{2}$$

The principal results concerning the stabilizing or destabilizing action of rotation have been considered in a number of papers (Bradshaw 1969; Johnston *et al.* 1972; Tritton 1981; Tritton & Davies 1981). The term 'stabilizing' (or 'destabilizing') is used here in the same general sense as it is used in discussion of stratified flow; i.e. one supposes that if the stability of a laminar flow is increased (decreased) by an external influence, then the corresponding turbulent flow is altered in structure to become in some sense less (more) vigorous. The main result is that, relative to the flow with $\Omega = 0$, the effect of rotation is destabilizing when

$$-1 < S < 0 \tag{3}$$

and stabilizing otherwise. Thus rotation in a sense such that S is positive is always stabilizing; rotation in the opposite sense is initially destabilizing but this trend is reversed with increasing rotation rate ('restabilization'). (Of course, in general, ζ and therefore S varies from place to place in a way that may itself be modified by changes in the turbulence structure. Depending on the context, the ideas about stabilization etc. may apply locally or in some averaged sense.) This may be summarized by introducing

$$B = S(1+S) \tag{4}$$

and saying that the sign and size of B indicate the stabilization or destabilization in a similar way to the Richardson number in a stratified flow (with positive sign for stabilization and negative for destabilization).†

However, the dynamical processes involved are less apparent than for stratified flow, mainly because the Coriolis force, unlike the buoyancy force, does not directly enter the turbulent energy balance equation; instead, it modifies the turbulence in such a way that the transfer of energy from the mean flow to the turbulence, by the process present for $\Omega = 0$, is weaker or stronger. Consequently, there is a greater need for heuristic models that support the above statements and clarify the likely consequences of the stabilizing or destabilizing processes. (The need for such support is also indicated by the fact that an over-simple (in my opinion) argument, leading to qualitatively similar but quantitatively different conclusions, has gained some currency (e.g. Lesieur, Yanase & Métais 1991). This supposes that destabilization

† There has recently been an increasing tendency to call B the 'Richardson number' – and even to drop the quotation marks. In my opinion the analogy is not a very close one and this is a misleading nomenclature. B requires its own name. May I suggest the Bradshaw number.

It is often convenient to refer to S rather than B, but it may be sufficient to call this the vorticity ratio or the reciprocal gradient Rossby number rather than giving it a special name.

arises from the cancellation of the rotational effect of the shear by that of the system rotation. It implies that maximum destabilization occurs when $S = -1$, and not $S = -\frac{1}{2}$ (maximum $-B$) (Tritton 1981).)

The role of B was first stated by Bradshaw (1969), but with rather little explanation. Although he must have been aware of some of the ideas below in order to reach his conclusions, these are not made explicit. Bradshaw refers particularly to the analogy with stratified flow and introduces, without derivation, a 'Brunt–Väisälä' frequency for rotating shear flow; this is real for $B > 0$ and imaginary for $B < 0$, and becomes just the intrinsic frequency of a rotating fluid, 2Ω, when $|S| \gg 1$.

Johnston *et al.* (1972) considered the production and Coriolis terms in the equations for the rates of change of each turbulence energy component and of the Reynolds shear stress. They supposed that the other terms in the equations, although not negligible, played comparatively little role in determining the relevant features of the turbulence structure. As discussed also by Tritton (1978), one can then see how the Coriolis terms alter that structure in a way that reduces or enhances the total energy transfer from the mean flow to the turbulence. This approach is referred to below as the simplified Reynolds stress equations (SRSE) scheme.

Tritton & Davies (1981) gave a simple 'displaced particle' argument showing the stabilizing or destabilizing role of Coriolis forces. This argument related, in the first place, to the stability of laminar flows. However, it may also be indicative of a process occurring within a turbulent flow.

The interpretation of any of these arguments must be consistent with the long-established result (e.g. Hide 1977) that a strictly two-dimensional flow is unaffected by rotation about an axis perpendicular to the planes of motion. More precisely, if a two-dimensional velocity field is a solution of the equations of motion in the absence of rotation, then that same velocity field is also a solution in a rotating system, for any value of the rotation rate. The pressure field is modified by rotation. It is not always immediately apparent why arguments predicting changes with changing rotation rate do not apply to two-dimensional motion, a point that has perhaps not been adequately mentioned in previous discussions.

We thus see that there are various contributions to our understanding of the action of rotation on turbulent shear flow, but that they are somewhat fragmented. Consequently, in addition to its primary purpose, explained above, of reformulating the SRSE scheme, this paper has a secondary purpose of showing some of the relationships between these contributions.

2. Displaced particle analysis

With this in mind, it is useful first to examine further the implications of the displaced particle argument of Tritton & Davies (1981). For convenience, the argument as given previously is briefly reproduced. Consider a shear flow as shown in figure 1 with positive $\mathrm{d}U/\mathrm{d}y$ and positive Ω so that S is negative, and suppose that a small perturbation leads to a fluid particle being displaced a small distance ϵ in the y-direction. (For ease of drawing, the particle is shown as a little blob. However, it should properly be thought of as a long thin rod in the x-direction; otherwise, longitudinal pressure gradients will arise in a way that invalidates the argument. Displaced and undisplaced 'rods' should then be thought of as at slightly different z, instead of the different x shown for the 'blobs'.) In its undisplaced position it had longitudinal velocity U_1 and when displaced it has velocity U_1'. There is a Coriolis force $2\rho\Omega U_1'$ acting on it as shown in figure 1 (ρ is fluid density). Similarly, a Coriolis

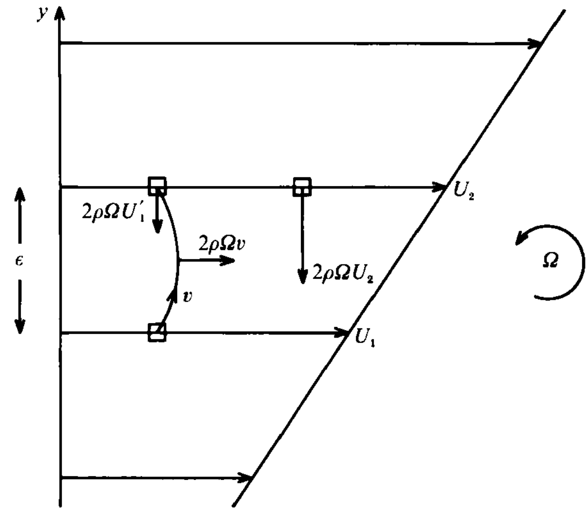

FIGURE 1. Forces on displaced and undisplaced 'particles': see text.

force $2\rho\Omega U_2$ acts on an undisplaced particle at its new position and this will be balanced by a pressure gradient in the y-direction. That pressure gradient also acts on the displaced particle. Hence, the net force tends to displace the particle further if $U_1' < U_2$ and to return it towards its original position if $U_1' > U_2$. $U_1' \neq U_1$ because of the Coriolis force that acted whilst the particle was moving with velocity v in the y-direction,

$$U_1' - U_1 = \int 2\Omega v \,\mathrm{d}t = 2\Omega\epsilon. \tag{5}$$

Hence,

$$U_1' - U_2 = 2\Omega\epsilon - \frac{\mathrm{d}U}{\mathrm{d}y}\epsilon = -\frac{\mathrm{d}U}{\mathrm{d}y}(S+1)\,\epsilon = \zeta(S+1)\,\epsilon. \tag{6}$$

Remembering that we are considering negative ζ (positive $\mathrm{d}U/\mathrm{d}y$) and negative S, we see that the particle tends to be displaced further if $S > -1$, restored if $S < -1$.

Corresponding considerations for positive S (Tritton & Davies 1981) show that the particle always tends to be restored in this case.

There is thus a tendency to instability through a mechanism, to which the shear and Coriolis effects are both intrinsic, occurring in the range $-1 < S < 0$. This tendency may be superimposed on any other instability mechanism, such as Kelvin–Helmholtz instability, that may be present even when $\Omega = 0$. We consider briefly below the implications for turbulence structure. First, however, we examine the consequences of the argument a stage further.

The net effect of the Coriolis and pressure forces on the displaced particle is a force

$$2\rho\Omega(U_2 - U_1') = -\rho\zeta^2 S(S+1)\,\epsilon. \tag{7}$$

If therefore we continue to consider, not a full fluid flow, but an isolated particle not interacting with other particles except in the way considered above,

$$\rho\frac{\mathrm{d}^2\epsilon}{\mathrm{d}t^2} = -\rho\zeta^2 S(S+1)\,\epsilon \tag{8}$$

and

$$\epsilon = \epsilon_0 \exp(\sigma t), \tag{9}$$

where σ is real when $-1 < S < 0$ and (choosing the positive root to correspond to the amplifying perturbation)

$$\sigma = -\zeta[-S(S+1)]^{\frac{1}{2}}. \tag{10}$$

σ is the magnitude of Bradshaw's (1969) 'Brunt-Väisälä frequency' (which is imaginary in this range).

Equation (9) implies

$$v = \sigma\epsilon_0 \exp(\sigma t). \tag{11}$$

Also, the x-component of the velocity of the particle relative to the basic flow is

$$u = U_1' - U_2 = \zeta(S+1)\,\epsilon_0 \exp(\sigma t) \tag{12}$$

from (6). Hence,

$$v/u = -[-S/(1+S)]^{\frac{1}{2}} = \tan\theta \tag{13}$$

say; i.e.

$$\sin\theta = \pm(-S)^{\frac{1}{2}} \quad (\text{with } \tan\theta < 0). \tag{14}$$

Thus the effect of the instability is that the particle tends to be accelerated away from its original position in a direction at an angle θ to the flow direction. This angle is of some significance in the discussions of §3, and we shall return to it there.

Any displaced particle argument, of course, gives only limited information about any actual fluid flow. However, some inferences about the consequences of the shear/Coriolis instability mechanism may be made. A displacement with either positive or negative ϵ is, of course, amplified. Also, motion in the z-direction is unaffected by Coriolis effects. Combined with the earlier remark that the particles should be considered as 'rods' in the x-direction, this suggests the generation of roll-like structures with their axes in the x-direction and circulation in planes at an angle θ to the x-direction. Various theoretical and experimental results on instabilities of laminar flows show such a structure. Most of these results are listed by Tritton & Davies (1981); some recent very striking examples have been given by Alfredsson & Persson (1989). In the present context of turbulent flow, the implication is that large eddies of a generally longitudinal roll-like character may arise spontaneously, superimposed on and interacting with the eddies from other eddy generating processes. Structures of this sort are prominent features in the observations of channel flow and boundary layers in a rotating fluid by Johnston *et al.* (1972) and Watmuff, Witt & Joubert (1985). Their role in a free shear layer is considered by Bidokhti & Tritton (1992).

3. Simplified Reynolds stress equation scheme

We come now to the central purpose of this paper, the reformulation of the approach, in terms of the energy component and Reynolds shear stress equations, of Johnston *et al.* (1972) (the SRSE scheme). The status and purpose of the scheme will be considered in the next section, but two comments should be made here. First, I have called it a 'scheme' rather than the more natural name of 'model', to avoid the implication that it has the status of an attempt to represent all aspects of a turbulent flow sometimes implied by the term 'turbulence modelling'. It has the more modest purpose of identifying key processes in the stabilization or destabilization of a shear flow by rotation. Secondly there are limitations to the applicability of the scheme that make it important to have a formulation in which one can check whether *and* when experimental observations correspond to the scheme.

One of the implications of the scheme is that the direction in which the turbulence is most intense (the orientation of the major principal axis of the Reynolds stress tensor) is altered by the Coriolis effect. Although this implication can be seen from the conventional formulation, it is informative to make it more explicit. We use coordinates that are arbitrarily (in the first place) oriented with respect to the flow direction; i.e. we define

$$x_1 = x\cos\alpha + y\sin\alpha; \quad x_2 = -x\sin\alpha + y\cos\alpha; \quad x_3 = z, \tag{15}$$

$$U_1 = U\cos\alpha; \quad U_2 = -U\sin\alpha, \tag{16}$$

$$u_1 = u\cos\alpha + v\sin\alpha; \quad u_2 = -u\sin\alpha + v\cos\alpha; \quad u_3 = w, \tag{17}$$

Because of the symmetry of the flow with respect to $\pm z$, the tensor axis rotation is confined to the (x, y)-plane. A range of π in α is needed and we choose $-\frac{1}{2}\pi < \alpha < \frac{1}{2}\pi$.

Standard procedures for determining turbulence energy and Reynolds stress equations lead to

$$\frac{\mathrm{D}\overline{u_1^2}}{\mathrm{D}t} = 2\zeta\overline{u_1^2}\cos\alpha\sin\alpha + 2\zeta\overline{u_1u_2}\cos^2\alpha + 4\Omega\overline{u_1u_2} + [\mathrm{OT}], \tag{18}$$

$$\frac{\mathrm{D}\overline{u_2^2}}{\mathrm{D}t} = -2\zeta\overline{u_2^2}\cos\alpha\sin\alpha - 2\zeta\overline{u_1u_2}\sin^2\alpha - 4\Omega\overline{u_1u_2} + [\mathrm{OT}], \tag{19}$$

$$\frac{\mathrm{D}\overline{u_3^2}}{\mathrm{D}t} = [\mathrm{OT}], \tag{20}$$

$$\frac{\mathrm{D}(\overline{u_1u_2})}{\mathrm{D}t} = -\zeta\overline{u_1^2}\sin^2\alpha + \zeta\overline{u_2^2}\cos^2\alpha + \Omega(\overline{u_2^2} - \overline{u_1^2}) + [\mathrm{OT}], \tag{21}$$

where, as above, $\zeta = -\mathrm{d}U/\mathrm{d}y$. In deriving these, Coriolis terms corresponding to rotation about the z-axis have been included, and it is still supposed that there is only one component of the mean velocity (in the old x-direction) varying in only one direction (the old y-direction). $\mathrm{D}/\mathrm{D}t$ denotes the rate of change following the mean flow. Putting $\alpha = 0$ gives the equations as previously formulated.

On the right-hand side of the equations, the only terms written explicitly are those corresponding to turbulence mean flow interaction and Coriolis effects. All other processes are lumped together as [OT] – 'other terms'. The implication is not that these terms are negligible (their role is considered further in §4). The idea is just that the primary effects of rotation can be perceived by considering the terms written explicitly in (18)–(21). For example, reverting for a moment to ($\alpha = 0, u_1 = u, u_2 = v$), if rotation modifies these terms so as to increase $\mathrm{D}\overline{v^2}/\mathrm{D}t$ and/or decrease $\mathrm{D}\overline{u^2}/\mathrm{D}t$, a likely consequence is an increase in $\overline{v^2}/\overline{u^2}$. Such modifications can come about not only through the terms specifically involving Ω but also through changes in the 'production' terms in turn arising from the changes in $\overline{u^2}$ etc. For further discussion in the context of the usual coordinates, see Tritton (1978).

In applying such ideas to the above equations, it is convenient to combine (18)–(20) to give

$$\frac{\mathrm{D}\overline{q^2}}{\mathrm{D}t} = \zeta[(\overline{u_1^2} - \overline{u_2^2})\sin 2\alpha + 2\overline{u_1u_2}\cos 2\alpha] + [\mathrm{OT}], \tag{22}$$

where
$$\overline{q^2} = \overline{u^2} + \overline{v^2} + \overline{w^2} = \overline{u_1^2} + \overline{u_2^2} + \overline{u_3^2}, \tag{23}$$

and to give

$$\frac{\mathrm{D}}{\mathrm{D}t}(\overline{u_1^2} - \overline{u_2^2}) = \zeta[(\overline{u_1^2} + \overline{u_2^2})\sin 2\alpha - 2\overline{u_1u_2}(1 + 2S)] + [\mathrm{OT}], \tag{24}$$

S being defined in equation (2). It is also convenient to rewrite (21) in the form

$$\frac{\mathrm{D}(\overline{u_1 u_2})}{\mathrm{D}t} = \tfrac{1}{2}\zeta[-(\overline{u_1^2}-\overline{u_2^2})(1+2S)+(\overline{u_1^2}+\overline{u_2^2})\cos 2\alpha]+[\mathrm{OT}]. \quad (25)$$

Two values of α are of particular significance. We call them α_a and α_b.

The first is the angle that makes x_1 and x_2 coincide with principal axes of the Reynolds stress tensor (x_3 does so automatically by symmetry). This implies that $\overline{u_1 u_2} = 0$ and so (22) and (24) become

$$\frac{\mathrm{D}\overline{q^2}}{\mathrm{D}t} = \zeta(\overline{u_1^2}-\overline{u_2^2})\sin 2\alpha_a+[\mathrm{OT}] \quad (26)$$

and

$$\frac{\mathrm{D}}{\mathrm{D}t}(\overline{u_1^2}-\overline{u_2^2}) = \zeta(\overline{u_1^2}+\overline{u_2^2})\sin 2\alpha_a+[\mathrm{OT}]. \quad (27)$$

Note that, both before and after this last stage, the equation for $\mathrm{D}\overline{q^2}/\mathrm{D}t$ did not contain a term explicitly involving S (and thus Ω); the equation for $\mathrm{D}(\overline{u_1^2}-\overline{u_2^2})/\mathrm{D}t$ did involve such a term before the last stage but does not do so after. None of these facts implies that rotation is playing no role in the equations; for example, in (26), both $(\overline{u_1^2}-\overline{u_2^2})$ and α_a will, in general, be affected by rotation.

In relating to principal axes it is convenient to define these more precisely so that $\overline{u_1^2} > \overline{u_2^2}$. We are already working to the convention that axes are chosen so that $\zeta < 0$. Hence, from (26), any behaviour for which the turbulence is extracting energy from the mean motion requires $\sin 2\alpha_a < 0$; i.e. $-\frac{1}{2}\pi < \alpha_a < 0$. This is saying no more than that $-\overline{uv}\,\partial U/\partial y$ must be positive, but it is a point that needs to be clear in the following discussion.

The other value of α that is of interest is the value that makes

$$\mathrm{D}(\overline{u_1 u_2})/\mathrm{D}t = 0 \quad (28)$$

when the [OT] are ignored; i.e. the value for which the mean flow interaction and Coriolis terms make no net contribution to the cross-correlation. From (25)

$$\cos 2\alpha_b = (\overline{u_1^2}-\overline{u_2^2})(1+2S)/(\overline{u_1^2}+\overline{u_2^2}). \quad (29)$$

The basic idea behind the SRSE scheme can be restated as saying that one supposes that

$$\alpha_a \approx \alpha_b, \quad (30)$$

i.e. that the orientation of the principal axes is governed by the orientation of the turbulence generating processes.

Stabilization or destabilization of the flow can be understood in the first place by considering $\alpha_a = \alpha_b$ exactly. Equations (26) and (27) indicate that there is a joint process in a shear flow by which $(\overline{u_1^2}-\overline{u_2^2})$ contributes to the generation of $(\overline{u_1^2}+\overline{u_2^2})$ (and so to $\overline{q^2}$), and $(\overline{u_1^2}+\overline{u_2^2})$ contributes to the generation of $(\overline{u_1^2}-\overline{u_2^2})$. The rate of each generation process is proportional to $-\sin 2\alpha_a$. One therefore anticipates that the turbulence energy generation process will be most efficient and so the turbulence most vigorous when $-\sin 2\alpha_a$ has its maximum value of 1; i.e. when $\alpha_a = -\frac{1}{4}\pi$ and $\cos 2\alpha_a = 0$. From (29) and the supposition that $\alpha_a = \alpha_b$, this occurs when

$$1+2S = 0, \quad (31)$$

which may be expressed as

$$2\Omega = \tfrac{1}{2}\partial U/\partial y. \quad (32)$$

Remembering that we are thinking in terms of $\partial U/\partial y$ being fixed and Ω, and so S, being varied and also that we are using the flow with $S = 0$ as the point of reference, one can say that maximum destabilization occurs at $S = -\frac{1}{2}$. The further S is from this value, the further the principal axis is rotated round from its optimum orientation of $\alpha_a = -\frac{1}{4}\pi$ and the less effective the turbulence generation process becomes. Thus, starting at $\Omega = 0$, increasing the rotation rate so that S is positive is always stabilizing, but increasing it so that S is negative is initially destabilizing and subsequently restabilizing.

How rapidly α_b (and so α_a according to the scheme) rotates as S is varied depends on $(\overline{u_1^2}-\overline{u_2^2})/(\overline{u_1^2}+\overline{u_2^2})$; i.e. on the anisotropy of the turbulence as measured by $\overline{u_1^2}/\overline{u_2^2}$. In the limiting case of $\overline{u_2^2} = 0$, α_b is identical with the angle θ introduced in §2 (equation (13)). The SRSE scheme then describes the type of motion discussed there. In turbulent motion, with $\overline{u_2^2} \neq 0$, α_b rotates with varying S more slowly than θ does. Equation (29) has a solution only when the right-hand side lies between -1 and 1. When S becomes too large, the scheme is bound to fail. However, provided $\overline{u_1^2}/\overline{u_2^2}$ is not too large, its range of potential applicability may extend well into the stabilized and restabilized regions (in contrast with the fact that θ is defined only for $-1 < S < 0$).

Information on the anisotropy has to be obtained empirically. This is not surprising since it is the case when $\Omega = 0$. The scheme does, however, indicate relative effects – i.e. the likely trend with varying rotation – in a way that may be compared with the empirical information.

Before considering this, it is useful to note that the scheme implies symmetry of developments about $S = -\frac{1}{2}$. In so far as the developments are determined by (26), (27) and (29) with the [OT] playing only a passive role and with $\alpha_a = \alpha_b$, the same value of $|S+\frac{1}{2}|$ implies the same values of $\sin 2\alpha_a$ (although opposite signs of $\cos 2\alpha_a$, and the same values of $(\overline{u_1^2}-\overline{u_2^2})/(\overline{u_1^2}+\overline{u_2^2})$. In particular, there is symmetry between $S = 0$ and $S = -1$.

Destabilization increases the anisotropy of the turbulence, in the sense that it increases $\overline{u_1^2}/\overline{u_2^2}$. This is most immediately apparent if we return to (18) and (19), which, for $\alpha = \alpha_a$ and so $\overline{u_1 u_2} = 0$, become

$$\frac{\mathrm{D}\overline{u_1^2}}{\mathrm{D}t} = \zeta\overline{u_1^2}\sin 2\alpha_a + [\mathrm{OT}], \tag{33}$$

$$\frac{\mathrm{D}\overline{u_2^2}}{\mathrm{D}t} = -\zeta\overline{u_2^2}\sin 2\alpha_a + [\mathrm{OT}]. \tag{34}$$

Since, as noted above, $\zeta \sin 2\alpha_a$ is positive, there is a transfer of energy from the mean motion to the energy of the component in the major principal axis direction and a smaller transfer to the mean motion from that in the minor principal axis direction. The anisotropy is thus self-perpetuating (but limited by the [OT] which include intercomponent transfer terms). This effect is strongest when $-\sin 2\alpha_a$ is maximum; i.e. around the peak of destabilization.

4. Status and purpose of the SRSE scheme

The SRSE scheme, in its previous formulation by Johnston *et al.* (1972), has played an important role in our understanding of shear flows in rotating fluids. It was successful, for example, in accounting for the observations of channel flow turbulence by Johnston *et al.* themselves. If and when it is applicable then it provides valuable insight into the basic dynamical processes. One naturally wants to turn to it in interpreting new experiments and in predicting features of the flow in yet further configurations. There are, however, significant queries about and limitations to its applicability, in addition to the fact that (29) has no solution when $|S|$ is too large.

These arise from the ignored [OT]. As mentioned in §3, the supposition is not that the [OT] are negligible, but only that they do not act in a way that causes marked departures from relationship (30). It is to be noted that, in the cases to be examined in §§7 and 8 (figures 2, 4 and 6; see also table 1 in §6.3 of Bidokhti & Tritton 1992), $\alpha_a \approx \alpha_b$ when $\Omega = 0$. One thus has a good starting point for using the SRSE scheme to understand the effect of varying Ω. It is, however, a necessary rather than a sufficient condition for so doing: the [OT] will vary with Ω, and may do so in ways that introduce bigger differences between α_a and α_b.

One process that is likely to be changed is the dissipation. For example, changes in this are an important aspect of the interpretation of the free shear layer observations in Bidokhti & Tritton (1992). It is plausible, however, that such changes do not reorient the larger-scale structure in a way that would make α_a and α_b differ. The implication is that the SRSE scheme will not account for all the effects of rotation but may still capture central aspects of the turbulence–mean flow interaction.

More problematic are the terms of the form $\overline{p(\partial u_i/\partial x_j + \partial u_j/\partial x_i)}$ (p being the pressure fluctuation), that, for example ($i = j$), transfer energy between components. In one respect, such terms are essential to the scheme: they provide energy for $\overline{w^2}(\equiv \overline{u_3^2})$ and changes in them permit $\overline{w^2}$ to be involved in the consequences of stabilization or destabilization (cf. (20)). Provided $\overline{w^2}$ shows similar trends to $(\overline{u^2}+\overline{v^2})$, this introduces no problem with the scheme (cf. the concept of $\overline{w^2}/\overline{u^2}$ 'following' $\overline{v^2}/\overline{u^2}$ in §§7.2 and 7.3 of Bidokhti & Tritton 1992). The problematic aspect is the possibility of redistribution within the (x, y)-plane, thus changing α_a. That this needs consideration is emphasized by considering two-dimensional turbulence.

As mentioned in §1, a two-dimensional turbulent motion will remain unchanged as the rotation rate varies. In practice this is a result of limited applicability, since two-dimensional turbulence (or an approximation to it) occurs only when three-dimensionality is suppressed by rapid rotation. The result only implies that, once one has reached this state, still further increase in the rotation rate will have no effect. This means that it is relevant only to large $|S|$. We have seen that the SRSE scheme then fails in any case because of the absence of a solution of (29). However, it raises an important point of principle. If one considers a hypothetical two-dimensional turbulent motion at low or moderate S, then the scheme would appear to apply and to predict changes. This failure of the scheme is due to the pressure–strain correlation terms that transfer energy between components. In a strictly two-dimensional motion, rotation modifies these in a way that exactly cancels out the corresponding transfer by Coriolis effects.

Failure of the scheme in this hypothetical situation raises the question of whether similar failure may occur in real situations. This is a major motivation for the examination of data, with which much of the rest of this paper is concerned, to see whether they accord with the scheme.

The SRSE scheme has similarities with rapid distortion theory. The relationship between the two is considered in Appendix A. The point is made there that the aims of the two are different, summarized by the words 'interpretative' and 'predictive'. This difference in aim is even more germane when one compares the SRSE scheme with turbulence models that develop approximations to every term in the equations (e.g. Cousteix & Aupoix 1982; Aupoix 1987; Lakshminarayana 1986; Warfield & Lakshminarayana 1987; Launder, Tselepidakis & Younis 1987; Andersson & Nilsen 1989*a*, *b*; Speziale 1989; Speziale, Sarkar & Gatski 1990). The former is very simplistic when compared with the latter. On the other hand, the very attempts at completeness in the latter may make it difficult to discern simplifying principles that aid our physical understanding of the dynamics. In this way the SRSE scheme complements the models. Indeed, the understanding it provides may promote improved modelling procedures. For it to do this, the assessment in the following sections of when it is applicable is important.

5. Application of the SRSE scheme to experimental data

α_a and α_b can both be calculated from the turbulence intensity components and the Reynolds shear stress that would normally be measured in an experimental investigation of a turbulent flow. Tests of the SRSE model by comparing them will be made in §§7 and 8. However, the most satisfactory way of making the comparison is not quite obvious and requires a brief discussion.

Evaluation of α_a is straightforward. From (15) and (17) along with the requirement that $\overline{u_1 u_2} = 0$, one can easily find that

$$\tan 2\alpha_a = 2\overline{uv}/(\overline{u^2}-\overline{v^2}). \tag{35}$$

One can also find that the corresponding value of $\overline{u_1^2}/\overline{u_2^2}$ is

$$\frac{\overline{u_1^2}}{\overline{u_2^2}} = \frac{\overline{u^2}\cos^2\alpha_a + \overline{v^2}\sin^2\alpha_a + \overline{uv}\sin 2\alpha_a}{\overline{u^2}\sin^2\alpha_a + \overline{v^2}\cos^2\alpha_a - \overline{uv}\sin 2\alpha_a}. \tag{36}$$

However, if $\alpha_a \neq \alpha_b$, this value of $\overline{u_1^2}/\overline{u_2^2}$ does not correspond to α_b. (The difference is explained more fully and illustrated in Appendix B.) We therefore denote by α_b' the angle given by substitution of this value of $\overline{u_1^2}/\overline{u_2^2}$ into (29). Moreover, the comparison is actually made between α_a and α_b' rather than between α_a and α_b. What this amounts to is the following. One is taking the anisotropy of the turbulence as an empirical starting point. One then asks, for this degree of anisotropy, what value of α_a does the hypothesis that $\alpha_a = \alpha_b$ predict, and compares the actual value of α_a with this prediction. It should be noted that equality of α_a and α_b' implies equality of α_a and α_b. Close agreement between α_a and α_b' may thus be interpreted as showing close agreement between α_a and α_b and thus a favourable test of the scheme.

There are two reasons for adopting the comparison between α_a and α_b'. First, we have seen in §3 that the scheme must fail when (29) ceases to have a solution. Working with α_b rather than α_b' provides a much less clear indication of this cut-off and leads to the danger that one attempts to interpret discrepancies between α_a and α_b in circumstances where comparison is meaningless. Secondly, although $\overline{u_1^2}/\overline{u_2^2}$ has to be obtained empirically, the scheme does predict trends in it, as discussed at the end of §3. One is thus evaluating α_b' in a way that makes use of partial prediction of it by the scheme. One then needs, of course, to check that $\overline{u_1^2}/\overline{u_2^2}$ is varying in the appropriate way.

6. An extension to the SRSE scheme

The following extension is generally relevant to developing flows in which S changes continuously with distance downstream. It has been developed, however, primarily in connection with the important question for whether the structure of the turbulence is ever changed by rotation to give total energy transfer from the mean motion to the turbulence; i.e. in the conventional formulation, whether Reynolds shear stress reversal occurs; or in the present formulation, whether the principal axes ever rotate outside the range $-\frac{1}{2}\pi < \alpha_a < 0$. In its simplest form, as above, the scheme never indicates such a development. As $|1+2S|$ is increased $|\cos 2\alpha_b'|$ increases. If it goes beyond unity, then (29) (with α_b replaced by α_b') just ceases to have a solution; it does not give a continuous development into, e.g., $\alpha_b' > 0$. (This inference can also be seen from the formulation by Johnston *et al.*, but not so immediately.)

We consider what happens when α_a is continuously changing, due principally to continuous changes in S. Changes in α_a might then, for example, lag behind those in α_b'. The axes can be chosen so that they are instantaneously the principal axes and $\overline{u_1u_2} = 0$, but (25) (even without the [OT]) may make $\mathrm{D}(\overline{u_1u_2})/\mathrm{D}t \neq 0$. It can be shown that, when $\overline{u_1u_2} = 0$,

$$\frac{\mathrm{D}\alpha_a}{\mathrm{D}t} = \frac{\mathrm{D}(\overline{u_1u_2})/\mathrm{D}t)}{(\overline{u_1^2}-\overline{u_2^2})}. \tag{37}$$

Substituting (25),

$$\frac{\mathrm{D}\alpha_a}{\mathrm{D}t} = -\tfrac{1}{2}\zeta[1+2S-\frac{\overline{u_1^2}+\overline{u_2^2}}{\overline{u_1^2}-\overline{u_2^2}}\cos 2\alpha_a]+[\mathrm{OT}]. \tag{38}$$

This may be rewritten as

$$\frac{\mathrm{D}\alpha_a}{\mathrm{D}t} = -\tfrac{1}{2}\zeta(1+2S)\left(1-\frac{\cos 2\alpha_a}{c}\right)+[\mathrm{OT}]. \tag{39}$$

When $|c| < 1$, then $c = \cos 2\alpha_b'$ and (39) may be interpreted as indicating how a small difference between α_a and α_b' causes α_a to evolve.

However, one may also consider the implications of (39) when $|c|$ is a little greater than 1, a situation in which the model has no 'equilibrium' solution with $\mathrm{D}\alpha_a/\mathrm{D}t = 0$ but may have an evolving one. This is probably not a meaningful point of view when $\cos 2\alpha_a$ and c are very different, but may be useful for small differences. In particular, we may apply it when $|\cos 2\alpha_a|$ is close to 1, and it then relates to the question of Reynolds shear stress reversal. We consider the case when α_a is close to 0, as may be expected from the considerations in §3 at large positive S. (The argument can be applied to α_a close to $-\frac{1}{2}\pi$ at large negative S with appropriate sign changes.) Then

$$\frac{\mathrm{D}\alpha_a}{\mathrm{D}t} = -\tfrac{1}{2}\zeta(1+2S)\left(1-\frac{1}{c}\right). \tag{40}$$

Reynolds stress reversal will be brought about by positive $\mathrm{D}\alpha_a/\mathrm{D}t$ so that α_a passes from negative values to positive. This corresponds to $c > 1$ (remembering that we always have $\zeta < 0$). This result will be used in §8.

7. Application of the SRSE scheme to homogeneous shear flow

Comparisons of α_a and α_b' may be made for the results of either numerical experiments or laboratory experiments. We consider the former here and the latter in the next section.

The only numerical experiments giving data in a form suitable for this treatment are those on a homogeneous shear flow by Bertoglio (1982) (see also Bertoglio *et al.* 1978; Bertoglio, Charnay & Mathieu 1980). He considers a uniform velocity gradient in a fluid rotating at rates such that S ranges from -0.875 to 0.875. The evolution of the turbulence from an initial condition of isotropy, is computed according to

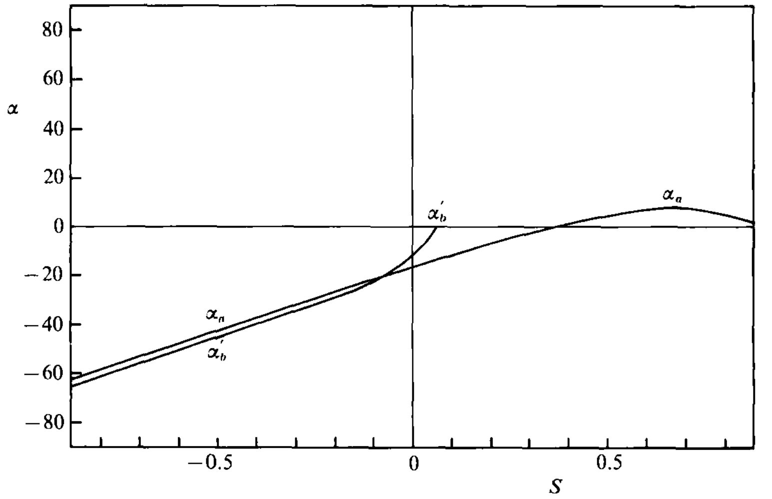

FIGURE 2. α_a and α_b' from numerical experiments on homogeneous shear flow (Bertoglio 1982).

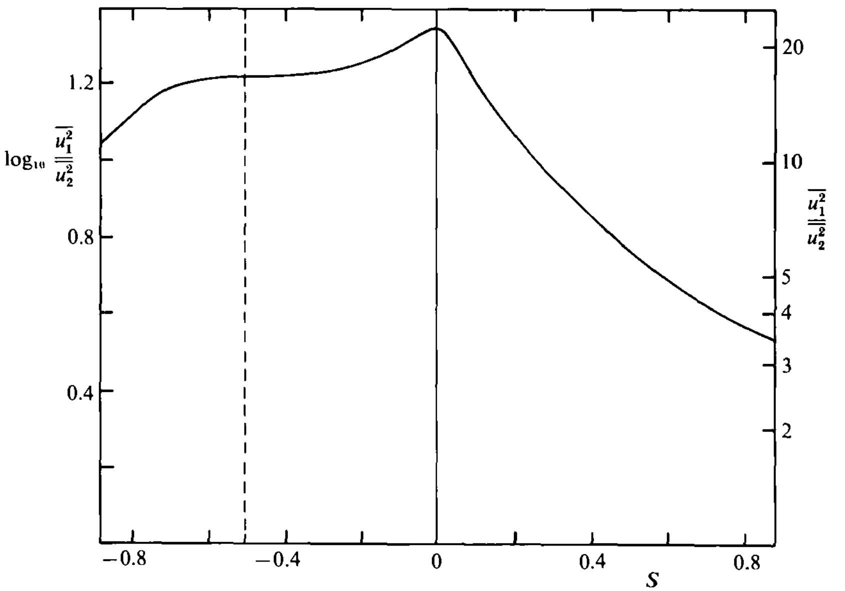

FIGURE 3. $\overline{u_1^2}/\overline{u_2^2}$ from numerical experiments on homogeneous shear flow. (Dashed line is at $S = -0.5$, where the model suggests that $\overline{u_1^2}/\overline{u_2^2}$ will be maximum.)

rapid distortion theory. In particular, data are given on the turbulence intensity components and the Reynolds stress.

From these α_a and $\overline{u_1^2}/\overline{u_2^2}$ have been evaluated using (35) and (36), and are plotted in figures 2 and 3. The value of $\overline{u_1^2}/\overline{u_2^2}$ was then used in (29) to give α_b' and this is also shown in figure 2. (Note that, on Bertoglio's graphs stabilizing rotation is on the left and destabilizing on the right, whereas the reverse convention is used here for consistency with that used in Bidokhti & Tritton (1992) – and the 'Richardson number convention' that positive corresponds to stabilization.)

This case has advantages and disadvantages for the present purpose. A major advantage is that S is uniform throughout the flow and perfectly known – in contrast with the laboratory experiments to be examined in §8.

On the other hand, the turbulence is evolving with time, and full data on intensities and the Reynolds stress are not given for other times. One does not know whether these would give similar results.

A more serious disadvantage becomes apparent from figure 3; the turbulence is very highly anisotropic in the sense that $\overline{u_1^2}/\overline{u_2^2}$ is large, greater than 10 throughout the range for which α_b' is evaluable and with a maximum greater than 20. (For $\Omega = 0$, this is a very significant discrepancy from laboratory results on homogeneous shear turbulence (Rose 1966). Applying (35) and (36) to the values at which the turbulence parameters equilibrate gives $\alpha_a = -31°$ and $\overline{u_1^2}/\overline{u_2^2} = 3.1$. The anisotropy is less before equilibration. Townsend (1976, p. 87) notes that a rapid distortion model overestimates the anisotropy.) This disadvantage has two consequences for the comparison of α_a and α_b'. First, the range of S for which (29) has a solution is restricted. It scarcely extends into the stabilized side, and so the model can work only for destabilization. (The data do not extend to restabilization, $S < -1$.) Secondly, α_b' differs little from the angle θ introduced in §2 (equation (13)). Consequently, the data do not distinguish much between the hypothesis that $\alpha_a \approx \alpha_b$ and the cruder hypothesis that $\alpha_a \approx \theta$; i.e. that the Reynolds stress tensor is directly oriented by the instability mechanism considered in §2.

With these qualifications, the closeness of the two curves on the left-hand side of figure 2 suggests that the model provides a good description of the primary effects of rotation, when these are destabilizing.

In considering the significance of this conclusion, we need to note some points in connection with the terms summarized by [OT] in (18)–(21). First, the homogeneity of the flow implies that terms representing transport of turbulence energy (or Reynolds stress) are absent. Nothing may be inferred about any role these terms might play in other flows. Secondly, since Bertoglio's computations are based on rapid distortion theory, transfer of energy between wavenumbers is excluded. Dissipation processes will not be correctly modelled for either non-rotating or rotating flow. One is thus assuming at the outset that these processes play a secondary role.

Thirdly, Bertoglio (1982) specifically presents (his figure 6) results on the pressure–strain correlation terms of equations (18)–(21) in conventional coordinates ($\alpha = 0$) and notes substantial changes due to rotation. One can thus assess directly the role of these terms. A full analysis will not be presented here, but two points may be made. First, the largest change is in the rate of energy transfer to $\overline{w^2}$. Such a change is to be expected if that component participates in the stabilization or destabilization, as noted in §4. Secondly, one may apply an appropriate coordinate transformation to the data in Bertoglio's figure 6 to determine the term under discussion in equation (21) for $\mathrm{D}\overline{u_1 u_2}/\mathrm{D}t$ when $\alpha = \alpha_a$. The results are consistent with the supposition that reorientation of the turbulence by this effect is relatively small. For example, over the range for which α_b' is definable, $\overline{p(\partial u_1/\partial x_2 + \partial u_2/\partial x_1)}$ averages about 0.3 times $\overline{p(\partial u/\partial y + \partial v/\partial x)}$.

8. Application of the SRSE scheme to a wake and a free shear layer

Laboratory experiments providing data with which the SRSE scheme may be compared are those on a wake by Witt & Joubert (1985) and those on a free shear layer by Bidokhti & Tritton (1990, 1992). The advantages and disadvantages of such flows for testing the scheme are just the reverse of those of the numerical experiments on homogeneous shear flow considered above. The advantages are first that none of

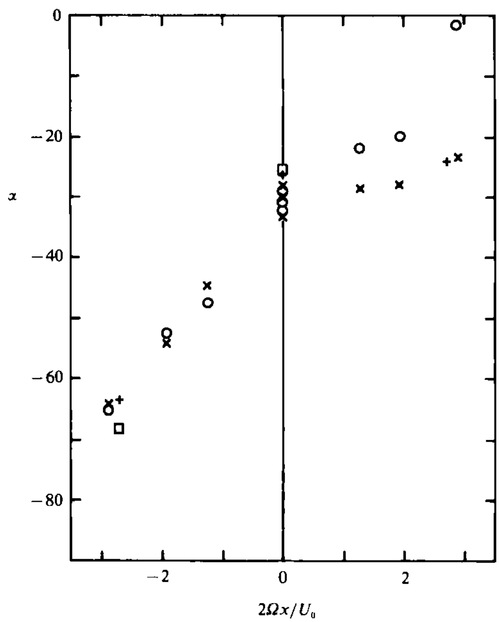

FIGURE 4. α_a (crosses) and α_b' (open symbols) from laboratory experiments on a wake. Derived from data in Witt & Joubert (1985) and Witt (1986) via figures 9 (*a*), 11 (*a*), and 16 of Bidokhti & Tritton (1992). × and ○: $2\Omega d/U_0 = 0.016$ (or 0); $x/d = 80$, 120, 180. +, □: $2\Omega d/U_0 = 0.0080$ (or 0); $x/d = 340$. Note: the measurements at $Q' = 2.72$ give no value of α_b' ($\cos 2\alpha_b' > 1$).

the 'other terms' are in principle zero and so the scheme is being more stringently tested; and secondly that the turbulence is not so highly anisotropic and so the range of potential applicability of the scheme is much greater. The disadvantages are that S both varies from place to place and has to be found from the experimental results (in a way that may give poor accuracy because it involves estimating the gradient of the mean velocity profile).

The spatial variations of S involve both variations with distance downstream and variations across the flow. The extension to the scheme in §6 was introduced to allow, at least in part, for the former. The variability of S across the flow is accompanied by variability of the other quantities $[\overline{v^2}/\overline{u^2}$ and $-\overline{uv}/(\overline{u^2}\,\overline{v^2})^{\frac{1}{2}}]$ involved in the evaluation of α_a and α_b'. The procedures adopted to give the results below are extensions of the data processing in Bidokhti & Tritton (1992). The reader wishing to know exactly how the quantities plotted in figures 4–6 have been derived will need to refer to that paper; this is the case for the wake as well as for the shear layer, because Witt & Joubert's data are reformulated there for comparison with the shear-layer data (see, in particular, Appendix A of Bidokhti & Tritton 1992 or the fuller presentation in Bidokhti & Tritton 1990). However, the simplifications involved amount to assuming that the structure of the turbulence is governed primarily by processes in the region where the turbulence energy production is strongest (other points being of minor significance with little effect on the outcome). This simplification is more severe for the wake than for the shear layer, because of the existence of another region of strong energy production, with the opposite sign of S, on the other side of the wake. Basically one is supposing that each side evolves independently of the other; comparison of the wake and shear layer in Bidokhti & Tritton (1992) suggests that this assumption is better than might be guessed.

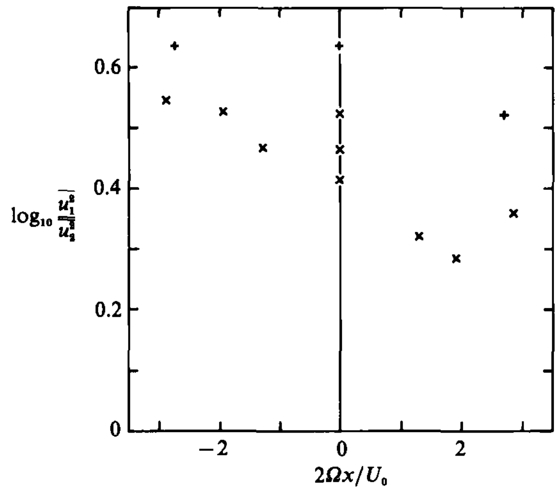

FIGURE 5. $\overline{u_1^2}/\overline{u_2^2}$ from laboratory experiments on a wake. Source and symbols as in figure 4.

Figures 4 and 6 show the comparison of α_a and α_b' for the two flows. In each case the abscissa is the similarity parameter used in Bidokhti & Tritton (1992) (with variations in this being produced mainly by measurements at different distances downstream in the case of the wake and by changes in the rotation rate in the case of the shear layer). Figure 16 of that paper relates S to Q and Q'. For the present purpose of comparing α_a and α_b', the main point is that, as in figure 2, the right-hand sides of figures 4, 5, and 6 correspond to stabilization and the left to destabilization and restabilization.

Figure 5 shows $\overline{u_1^2}/\overline{u_2^2}$, calculated on the route to α_b' but of some interest in its own right, for the wake. The corresponding plot for the shear layer is figure 13 of Bidokhti & Tritton (1992); the main features relevant to the following are that $\overline{u_1^2}/\overline{u_2^2}$ has a maximum of about 4.5 at about $Q = -5$ and that it falls particularly rapidly as Q becomes more negative than this, with the turbulence being almost isotropic when $Q < -15$.

Table 1 of Bidokhti & Tritton (1992) lists results of previous shear-layer experiments with $\Omega = 0$ for comparison purposes. The table includes quantities relevant to the present considerations. (α_b is listed as well as α_b'; the reason for preferring the latter is not so relevant here.)

The wake results are shown as individual points, since there were relatively few measurements, with crosses in figure 4 representing α_a and open symbols α_b'. The correspondence between α_a and α_b' on the left-hand side of figure 4 (the destabilized and incipient restabilized range) is remarkably close considering the way the scheme has been applied (ignoring variations of S across the wake). It should be remembered, however, that the scheme also implies a maximum in $\overline{u_1^2}/\overline{u_2^2}$ when $\alpha_a = -45°$ ($2\Omega x/U_0 = -1.2$) and this is not conspicuous in figure 5.

Because of the much larger amount of data, with considerable scatter, the shear-layer results are represented by smoothed curves. The smoothing was actually carried out on the original quantities from which α_a and α_b' are calculated. (Fuller details of the procedure are available in Bidokhti & Tritton 1990.) Thus the α_a curve in each part of figure 6 is effectively a smoothing of figure 12 of Bidokhti & Tritton (1992), though not actually obtained directly from it. Figures 6(*a*) and 6(*b*) differ in the way α_b' has been evaluated, specifically in the value of S (for use in (29)) taken

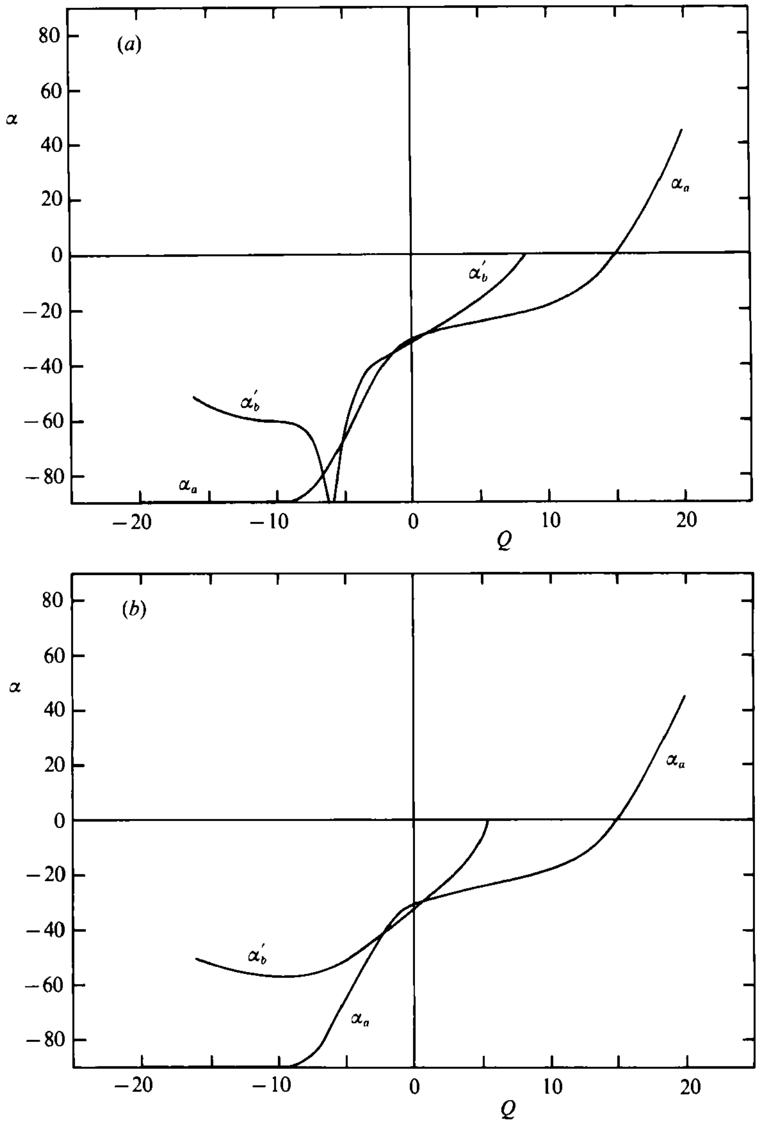

FIGURE 6. α_a and α_b' from laboratory experiments on a free shear layer. (*a*) Curves derived from smoothing of figures 9 (*a*) and 11 (*a*) and curve in figure 16 of Bidokhti & Tritton (1992). (*b*) Curves derived as in (*a*) except that $S_0 = 0.14Q$ used instead of curve in figure 16.

at each value of Q. Figure 6(*a*) uses the 'best estimate' of $S_0(Q)$ as shown in figure 16 of Bidokhti & Tritton (1992), and may therefore be considered the best available comparison between α_a and α_b'. However, as noted in the other paper, the experimental uncertainties in the determination of S_0 are considerable and it is not certain that all the details of figure 16 are significant. Consequently, the analysis has also been made using the simplest possible relationship of $S_0 \alpha Q$ (with the constant of proportionality taken at the average value of 0.14); this corresponds to ignoring the 'feedback effect' of changes in the mean velocity profile resulting from the changed structure of the turbulence. The upshot is figure 6(*b*).

The differences between figures 6(*a*) and 6(*b*) indicate a need for caution in the following discussion of the former. (The differences can be viewed more positively if one is prepared to interpret the data assuming the validity of the SRSE scheme, rather than trying to use it to test the validity. One can then argue that the better agreement of figure 6(*a*) indicates what processes are important in determining the structure of the turbulence – as represented by α_a. This aspect is considered in §§7.1 and 7.5 of Bidokhti & Tritton 1992.†) Figure 6(*a*) shows close correspondence between α_a and α_b' for a region covering the destabilized range and extending into the stabilized and restabilized ranges. Also $\overline{u_1^2}/\overline{u_2^2}$ has a maximum in the destabilized range – see above – as implied by the scheme. We consider the large-positive-Q behaviour below. When Q becomes negatively large, the behaviour of α_b' becomes complicated. The formal algebra gives the curves shown. In figure 6(*a*), α_b' ceases to exist ($\cos\alpha_b' < -1$) for a short range (although only a slightly different choice of the original smoothed curves would eliminate this range). It then reappears and increases. The initial rapid increase is associated with a region in which $-S_0$ decreases as $-Q$ increases (figure 16 of Bidokhti & Tritton 1992). The subsequent more gradual increase, also seen in figure 6(*b*), derives from rapid decrease of $\overline{u_1^2}/\overline{u_2^2}$. One would probably not expect α_a to follow α_b' in this respect; i.e. for rotation of the principal axes of the Reynolds stress tensor to actually be reversed as $-Q$ increases. In fact α_a remains around $-90°$ (although it also becomes increasingly ill-defined as the turbulence becomes nearly isotropic).

Section 6 has described an extension to the scheme that allows for flow development. In most respects the data do not warrant considering this refinement, but it does aid interpretation of the behaviour of the shear layer at large $|Q|$, particularly the difference between positive and negative Q. For the former α_a becomes positive (corresponding to reversal of the Reynolds shear stress from its normal sign). We have seen that the simplest form of the scheme does not allow such reversal (but just fails) whereas the extended form does. In fact, on the positive Q side, once α_b' has ceased to exist, $c > 1$ (c being defined in §6) and increase of α_a through 0 is to be expected. It occurs, although at a value of Q significantly larger than where c becomes greater than 1. For negative Q, in contrast, the features described in the previous paragraph imply that c does not become less than -1 (except possibly for a short range). The extended scheme does not suggest the occurrence of Reynolds stress reversal (α_a decreasing through $-90°$) and, in fact, it does not occur.

9. Concluding remarks

This paper originated in an attempt to clarify and synthesize the previously rather fragmented concepts underlying stabilization, destabilization, and restabilization and the role of the parameter B in specifying these. It has developed in a way that has made the SRSE scheme far the largest part of this clarification, and this is the part requiring some rounding off.

The improvements to our understanding of the SRSE scheme have a negative aspect and a positive aspect. The former is that limitations to its applicability – the fact that α_b' does not exist for large $|S|$ and the points considered in §4 – that had

† Ideally one would prefer either that the data were clearcut enough to test the scheme unambiguously or that the scheme was sufficiently well established for it to be confidently used in interpreting the data. In fact one is forced to attempt both simultaneously. Hence, optimistically, agreement lends support to both the scheme and the interpretation; or, pessimistically, is fortuitous.

perhaps not previously been fully recognized have been made explicit. The latter is the formulation and use of a procedure to identify when the scheme is a good representation of the effect of rotation on the turbulence–mean flow interaction.

The results of this use are conveyed by figures 2, 4, and 6. The similarity of the trends shown by the three flows is notable and suggests that the results are of some generality. All three flows show close correspondence between α_a and α_b' – and thus the behaviour indicated by the scheme – in the destabilized range (conveniently identified in this context as the range in which $|\sin 2\alpha_a|$ is larger than its value for $S = 0$). The similarity of the trends continues into the stabilized region, except that in the case of figure 2 the abnormally high anisotropy results in the quick termination of the α_b' curve. As S increases, significant differences between α_a and α_b' appear; in each case the former varies more slowly than the latter. The qualitative prediction that rotation reduces $-\alpha_a$ is fulfilled, but quantitative correspondence with the scheme becomes poorer. One is then approaching the region in which the limitations mentioned above become operative and the scheme must lose applicability.

On the negative-S side, only the shear layer data extend into the restabilized range. Use has been made of the scheme, in conjunction with other considerations, in interpreting the behaviour here (in §8 and in Bidokhti & Tritton 1992). However, in view of the fact that it is a single case, and one involving considerable complications, it may be premature to draw any general conclusions. This is the range for which there is the greatest need for further experiments on various flows.

Appendix A. The SRSE scheme and rapid distortion theory

There are evident similarities between the SRSE scheme and rapid distortion theory (RDT) (Savill 1987 and references therein). This Appendix aims at clarifying the relationship between the two.

In a sense the SRSE scheme is a yet further simplification of the equations additional to the simplifications made in RDT. The latter also ignores many of the terms denoted [OT] in equations (18)–(21). It does, however, partially include the pressure–strain correlation term responsible for transfer of energy between components. RDT thus takes into account a process that we have noted as one of the reasons for the limitations of the SRSE scheme.

However, the two formulations are rather different in their points of view. They contribute in different, complementary, ways to our overall understanding. The difference concerns the way each formulation is used and the purpose of so using it. It may be summarized by saying that RDT is 'evolutionary' and 'predictive', whilst the SRSE scheme is 'quasi-equilibrium' and 'interpretative'. RDT is usually used to calculate, subject to various approximations and assumptions, the development of the structure of shear flow turbulence from some supposed initial state, such as isotropic turbulence. Its predictions may be compared (with varying degrees of success) with observed structure. It has extensively used for non-rotating shear flows (e.g. Townsend 1970, 1980) and its use for rotating flow (Bertoglio 1982) extends its applications. The SRSE scheme in contrast takes the structure of the non-rotating flow as an empirical starting point and then attempts to provide understanding of the way this is modified by rotation. In so doing, it assumes that the structure is largely governed by local conditions – the 'quasi-equilibrium' aspect (although the extension in §6 relaxes this to some extent). The equations are not solved but used to analyse data and so to see whether the effect of rotation on the turbulence–mean flow interaction can be interpreted in this way.

Appendix B. Why $\alpha_b' \not\equiv \alpha_b$

It was noted in §5 that the quantity α_b' plotted in figures 2, 4, and 6 differs from α_b, defined as the value of α that makes the right-hand side of (21) equal to zero when the [OT] are ignored. To clarify this distinction, table 1 lists a selection of the data underlying figure 6(*b*), and various quantities calculated from the data. This case has been chosen partly because the assumed proportionality of S to Q makes it easier to relate table and figures; but mainly because points with poor correspondence between α_a and α_b' are needed to illustrate the difference between α_b and α_b'.

Column 1 lists the values of S selected for this illustration; as explained in §8, S is here taken as $S = 0.14Q$.

Columns 2 and 3 list the observed quantities (based on smoothing of data in Bidokhti & Tritton 1992) from which subsequent columns are calculated. These columns are labelled with '$\alpha = -90°$' (making $u_1 = -v$ and $u_2 = u$) for consistency with the labelling of columns 5, 6, 8, and 9 and thus as an aid to seeing the relationships between the various quantities.

Column 4 lists values of α_a, the orientation of the major principal axis of the Reynolds stress tensor, evaluated from (35). The ratio of the major to minor principal axes is then given by (36) and is listed in column 5. The entries in column 6 are zero by definition; the column is nevertheless included for direct comparison with columns 3 and 9.

From its definition as the value of α that makes the explicit terms on the right-hand side of (21) equal to zero, α_b may be evaluated by writing $\overline{u_1^2}$ and $\overline{u_2^2}$ in terms of $\overline{u^2}, \overline{v^2}$, $\overline{uv}$ and α_b. This gives

$$\tan 2\alpha_b = \frac{\overline{v^2}(1+S) - \overline{u^2}S}{\overline{uv}(1+2S)}. \tag{B 1}$$

Values so obtained are given in column 7, and related quantities in columns 8 and 9; e.g. column 8 is obtained using (36) but with α_b replacing α_a. (Columns 8 and 9 are of little significance for the structure of the turbulence, other than for the purpose of the present illustration.)

Column 10 gives values of the right-hand side of (29) (denoted c, cf. §6), and column 11 the consequent values of α_b'. The last two lines have been selected as lying on either side of the point at which α_b' ceases to exist. The reason that α_b' is not identical with α_b is that the value of $\overline{u_1^2}/\overline{u_2^2}$ used in (29) is the quantity in column 5, not that in column 8. The two are the same when $\alpha_a = \alpha_b$ and so then $\alpha_a = \alpha_b'$ also. The cases in the table have been selected to include some for which α_a and α_b are close, and therefore the entries in columns 5 and 8 are close, the entry in column 9 is small, and α_b' is also close to α_a; and some for which the departures are substantial.

Hence, the hypothesis that $\alpha_a \approx \alpha_b$ can be tested by comparing α_a and α_b'. The reasons for choosing the latter comparison are given in §5.

When there are significant departures from $\alpha_a \approx \alpha_b$, α_b and α_b' tend to be closer to one another than either is to α_a. Equality of or a difference between α_b and α_b' is not itself necessarily a significant indicator; for example, when $S = -0.5$, α_b and α_b' are automatically equal to one another at $-45°$. When $|c| > 1$ and so α_b' is undefined, then one cannot have $\alpha_a = \alpha_b$. (The fact that the model must fail at large $|S|$ can be seen in terms of the behaviour of α_b as well as that of α_b'. If one formally puts $|S| \to \infty$ in (36), one gets

$$\tan 2\alpha_a \tan 2\alpha_b = -1 \tag{B 2}$$

and there is no possibility that $\alpha_a = \alpha_b$. What is much more complicated to see in this

S	$\alpha = -90°$: $\overline{v^2}/\overline{u^2}$	$\alpha = -90°$: $-\overline{uv}/(\overline{u^2}\,\overline{v^2})^{\frac{1}{2}}$	α_a (deg.)	$\alpha = \alpha_a$: $\overline{u_1^2}/\overline{u_2^2}$	$\alpha = \alpha_a$: $\overline{u_1 u_2}/(\overline{u_1^2}\,\overline{u_2^2})^{\frac{1}{2}}$
−0.84	2.25	0.25	−74.5	2.65	0
−0.56	1.68	0.525	−58.5	3.6	0
−0.28	0.83	0.46	−39.5	2.75	0
0	0.655	0.40	−31	2.6	0
0.28	0.61	0.365	−28	2.5	0
0.70	0.585	0.31	−24	2.3	0
0.84	0.585	0.29	−23.5	2.25	0
Column 1	2	3	4	5	6

S	α_b (deg.)	$\alpha = \alpha_b$: $\overline{u_1^2}/\overline{u_2^2}$	$\alpha = \alpha_b$: $\overline{u_1 u_2}/(\overline{u_1^2}\,\overline{u_2^2})^{\frac{1}{2}}$	c	α_b' (deg.)
−0.84	−51	1.9	−0.34	−0.305	−56
−0.56	−47	3.2	−0.255	−0.068	−47
−0.28	−39	2.75	−0.005	0.205	−39
0	−32	2.6	0.02	0.44	−32
0.28	−24	2.5	−0.06	0.67	−24
0.70	−13.5	2.15	−0.16	0.95	−9
0.84	−11	2.1	−0.175	1.03	—
Column	7	8	9	10	11

TABLE 1. Examples of quantities relating to figure 6(*b*): see text. (Note that the figures are quoted with more rounding than was used in the calculation; they may not check exactly.)

formulation is that decreasing anisotropy allows the model to be applied, with some prospect of success, to higher values of $|S|$. Calculation of α_b when α_b' does not exist tends to lead to erratic behaviour. In these circumstances, α_b is probably a quantity of little significance for the structure of the turbulence.)

REFERENCES

ALFREDSSON, P. H. & PERSSON, H. 1989 *J. Fluid Mech.* **202**, 543.

ANDERSSON, H. I. & NILSEN, P. J. 1989*a* *Norwegian Inst. Tech., Div. Appl. Mech., Rep.* 89:01.

ANDERSSON, H. I. & NILSEN, P. J. 1989*b* *ASME Fluids Engng Spring Conf., San Diego: Forum on Turbulent Flows.*

AUPOIX, B. 1987 *AGARD Rep.* 755, 3–1.

BERTOGLIO, J. P. 1982 *Am. Inst. Aero. Astro. J.* **20**, 1175.

BERTOGLIO, J. P., CHARNAY, G., GENCE, J. N. & MATHIEU, J. 1978 *C.R. Acad. Sci. Paris* A **286**, 957.

BERTOGLIO, J. P., CHARNAY, G. & MATHIEU, J. 1980 *J. Méc. Appl.* **4**, 421.

BIDOKHTI, A. A. & TRITTON, D. J. 1990 *University of Newcastle upon Tyne, Dept. Phys., Rep.* GFD 90/1.

BIDOKHTI, A. A. & TRITTON, D. J. 1992 *J. Fluid Mech.* **241**, 469.

BRADSHAW, P. 1969 *J. Fluid Mech.* **36**, 177.

COUSTEIX, J. & AUPOIX, B. 1982 Rotation and curvature effects on Reynolds stresses in boundary layers. *ONERA Rep.*

HIDE, R. 1977 *Q. J. R. Met. Soc.* **103**, 1.

JOHNSTON, J. P., HALLEEN, R. M. & LEZIUS, D. K. 1972 *J. Fluid Mech.* **56**, 533.

LAKSHMINARAYANA, B. 1986 *Am. Inst. Aero. Astro. J.* **24**, 1900.

LAUNDER, B. E., TSELEPIDAKIS, D. P. & YOUNIS, B. A. 1987 *J. Fluid Mech.* **183**, 63.

LESIEUR, M., YANASE, S. & MÉTAIS, O. 1991 *Phys. Fluids* A **3**, 403.

ROSE, W. G. 1966 *J. Fluid Mech.* **25**, 97.

SAVILL, A. M. 1987 *Ann. Rev. Fluid. Mech.* **19**, 531.

SPEZIALE, C. G. 1989 *Theoret. Comput. Fluid Dyn.* **1**, 3.

SPEZIALE, C. G., SARKAR, S. & GATSKI, T. B. 1990 *NASA Contractor Rep., ICASE Rep.* 90-5.

TOWNSEND, A. A. 1970 *J. Fluid Mech.* **41**, 13.

TOWNSEND, A. A. 1976 *The Structure of Turbulent Shear Flow.* 2nd edn. Cambridge University Press.

TOWNSEND, A. A. 1980 *J. Fluid Mech.* **98**, 171.

TRITTON, D. J. 1978 In *Rotating Fluids in Geophysics* (ed. P.H. Roberts & A. M. Soward), p. 105. Academic.

TRITTON, D. J. 1981 *Phys. Fluids* **24**, 1921.

TRITTON, D. J. & DAVIES, P. A. 1981 In *Hydrodynamic Instabilities and the Transition to Turbulence* (ed. H. L. Swinney & J. P. Gollub), p. 229. Springer.

WARFIELD, M. J. & LAKSHMINARAYANA, B. 1987 *Am. Inst. Aero. Astro. J.* **25**, 957.

WATMUFF, J. H., WITT, H. T. & JOUBERT, P. N. 1985 *J. Fluid Mech.* **157**, 405.

WITT, H. T. 1986 Effects of rotation on turbulent boundary layers and wakes. Ph.D. thesis, University of Melbourne.

WITT, H. T. & JOUBERT, P. N. 1985 *Proc. 5th Symp. on Turbulent shear flows, Cornell Univ.*, p. 21. 25.